

# Coherent bremsstrahlung at relativistic heavy ion colliders

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## Abstract

Coherent bremsstrahlung (CBS) is a specified type of radiation at colliders with short bunches. In the present paper I calculate the main characteristics of CBS for the LHC collider (in the PbPb mode) and for the RHIC collider. At this colliders  $dN_\gamma \sim N_0 dE_\gamma / E_\gamma$  photons of CBS will be emitted for a single collision of the beams in the energy range  $E_\gamma \lesssim E_c$ , where for the LHC collider (in the PbPb mode)  $N_0 = 50$ ,  $E_c = 90$  eV and for RHIC collider  $N_0 = 600$ ; 50 and 170,  $E_c = 0.08$ ; 0.5 and 0.8 eV for AuAu, pAu and pp collisions respectively.

It seems that CBS can be a potential tool for optimizing collisions and for measuring beam parameters.

Indeed, the bunch length  $\sigma_z$  can be found from the CBS spectrum because critical energy  $E_c \propto 1/\sigma_z$ ; the transverse bunch size  $\sigma_\perp$  is related to  $dN_\gamma \propto 1/\sigma_\perp^2$ . Besides, CBS may be very useful for a fast control over an impact parameter  $R$  between the colliding bunch axes because a dependence of  $dN_\gamma$  on  $R$  has a very specific behavior.

## 1 Three types of radiation at colliders

Let us speak, for definiteness, about emission by ion with a charge  $Z_1$  moving through a bunch of ions with the charge  $Z_2$ . If the photon energy is large enough, one deals with the ordinary (incoherent) *bremsstrahlung*.

If the photon energy becomes small enough, the radiation is determined by the interaction of the ion  $Z_1$  with the collective electromagnetic field of the second bunch. It is known (see, e.g. §77 in ref. [1]) that the properties of this coherent radiation are quite different depending on whether the deflection angle  $\theta_d$  of ion  $Z_1$  is large enough or rather small as compared with the typical emission angle<sup>1</sup>  $\theta_r \sim 1/\gamma_1$ . Let us give an estimate

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<sup>1</sup>We denote the following:  $N_1$  and  $N_2$  are the numbers of particles in the bunches;  $\sigma_z$  is the longitudinal,  $\sigma_x$  and  $\sigma_y$  are the horizontal and vertical transverse sizes of the second bunch;  $\gamma_1 = E_1/m_1 c^2$  is the Lorentz factor of ion  $Z_1$ ;  $E_c = 4\gamma_1^2 \hbar c / \sigma_z$  is the characteristic (critical) energy for the coherent bremsstrahlung photons;  $r_1 = (Z_1 e)^2 / m_1 c^2$  is the classical radius of  $Z_1$  ion.

for the deflection angle.

Electric  $\mathbf{E}$  and magnetic  $\mathbf{B}$  fields of the second bunch are approximately identical in magnitude,

$$|\mathbf{E}| \approx |\mathbf{B}| \sim \frac{Z_2 e N_2}{\sigma_z \sigma_x}, \quad (1)$$

transverse and deflect the ion  $Z_1$  in the same direction. In such fields the ion  $Z_1$  moves over circumference of the radius  $\rho \sim \gamma_1 m_1 c^2 / (Z_1 e B)$  and bents on the angle  $\theta_d \sim \sigma_z / \rho$ . It is also useful to introduce the length  $l_\rho = \rho / \gamma_1 \sim m_1 c^2 / (Z_1 e B)$  on which the ion  $Z_1$  would deflect at the angle  $1/\gamma_1$ . As a result, it is appeared a dimensionless parameter  $\eta$  such as

$$\frac{\theta_d}{\theta_r} \sim \frac{\sigma_z}{l_\rho} \sim \eta; \quad \eta = \frac{Z_2}{Z_1} \frac{r_1 N_2}{\sigma_x}. \quad (2)$$

We call a second bunch *long* if  $\sigma_z \gg l_\rho$  or  $\eta \gg 1$ . The radiation in this case is usually called *beamstrahlung*. Its properties are similar to those for the ordinary synchrotron radiation in an uniform magnetic field (see, e.g. review [2]).

We call a second bunch *short* if  $\sigma_z \ll l_\rho$  or  $\eta \ll 1$ . In this case the motion of the ions can be assumed to remain rectilinear over the course of the collision. The radiation in the field of a short bunch differs substantially from the synchrotron one. In some respect it is similar to the ordinary bremsstrahlung, which is why we called it *coherent bremsstrahlung* or CBS.

For the LHC (in the mode PbPb) and for the RHIC colliders the parameter  $\eta$  is of the order of

$$\eta \sim 10^{-4} - 10^{-3}, \quad (3)$$

so these colliders are colliders with short bunches.

In most colliders either  $\eta \ll 1$  (all the  $p\bar{p}$ ,  $\bar{p}p$  and relativistic heavy-ion colliders, some  $e^+e^-$  colliders and some B-factories) or  $\eta \sim 1$  (e.g., LEP, TRISTAN, some B-factories) and only the linear  $e^+e^-$  colliders have  $\eta \gg 1$ . Therefore, the CBS has a very wide region of applicability.

A classical approach to CBS was given in ref. [3]. A quantum treatment of CBS and some applications of CBS to the modern and future colliders were given recently in refs. [4–8]. The first consideration of CBS for relativistic heavy-ion colliders was performed in ref. [7].

## 2 Distinctions of the CBS from the usual bremsstrahlung and from the beamstrahlung

For the usual bremsstrahlung the number of photons emitted by ions  $Z_1$  is proportional to the number of ions in the first and second bunches:

$$dN_\gamma \propto N_1 N_2 \frac{dE_\gamma}{E_\gamma}. \quad (4)$$

In contrast, the number of CBS photons

$$dN_\gamma \propto N_1 N_2^2 \frac{dE_\gamma}{E_\gamma} \quad \text{at} \quad E_\gamma \lesssim E_c = \frac{4\gamma_1^2 \hbar c}{\sigma_z}. \quad (5)$$

Indeed, when a photon energy decreases, the coherence length  $\sim 4\gamma_1^2 \hbar c / E_\gamma$  becomes comparable to the length of the second bunch,  $\sigma_z$ , and at  $E_\gamma \lesssim E_c$  radiation is caused by the interaction of an ion  $Z_1$  with the second bunch as a whole, but not with each ions  $Z_2$  separately. In these conditions the second bunch is similar to a “particle” with the charge  $Z_2 e N_2$  and with an internal structure described by a form factor of the bunch. As a result, a probability of radiation is proportional to  $N_2^2$  and number of the emitted photons is proportional to  $N_1 N_2^2$ .

CBS differs from the beamstrahlung first of all due to the soft part of its spectrum. In this region for CBS (as well as for the ordinary bremsstrahlung)  $dN_\gamma \propto dE_\gamma / E_\gamma$ . Therefore, the total number of CBS photons is infinite in contrast to beamstrahlung for which (as well as for synchrotron radiation) the total number of photons is finite.

### 3 Experimental status and applications

The experimental status of the above-mentioned types of radiation is quite different.

The ordinary bremsstrahlung is a well-known process. At  $e^+e^-$  and  $ep$  colliders its cross section large enough and it often is an unwanted background. On the other hand, its large cross section and small angular spread of photons allows one to use this radiation for measuring one of the important parameter of a collider — luminosity (for example, at the VEPP-2M, HERA and LEP colliders).

The beamstrahlung was observed in a single experiment at SLC (ref. [9]), where it was shown that it can be used for measuring transverse bunch size.

The main characteristic of CBS was calculated only recently, experiments for its observation are planned in Novosibirsk, Cornell and Fermilab, but have not yet been performed. Therefore, one can speak about applications of CBS on the preliminary level only. Nevertheless, even now we can see such features of CBS which can be useful for applications. They are the following.

A huge number of the soft photons whose spectrum is determined by the length of the second bunch are emitted. The number of CBS photons for *a single collision* of the beams is

$$dN_\gamma = N_0 \Phi(E_\gamma / E_c) \frac{dE_\gamma}{E_\gamma}. \quad (6)$$

Here for the round Gaussian bunches, e.g. at

$$\sigma_{1x} = \sigma_{1y} \equiv \sigma_1; \quad \sigma_{2x} = \sigma_{2y} \equiv \sigma_2,$$

constant  $N_0$  is equal to

$$N_0 = \frac{4}{3\pi} \alpha N_1 \left( \frac{Z_2 r_1 N_2}{\sigma_1} \right)^2 \ln \frac{(\sigma_1^2 + \sigma_2^2)^2}{2\sigma_1^2 \sigma_2^2 + \sigma_2^4}, \quad (7)$$

and for the round and identical Gaussian bunches, i.e. at

$$\sigma_1 = \sigma_2 \equiv \sigma_\perp,$$

it is

$$N_0 = \frac{4}{3} \ln \frac{4}{3} \frac{\alpha}{\pi} N_1 \left( \frac{Z_2 r_1 N_2}{\sigma_\perp} \right)^2 = 0.89 \cdot 10^{-3} N_1 \left( \frac{Z_2 r_1 N_2}{\sigma_\perp} \right)^2. \quad (8)$$

The function

$$\Phi(x) = \frac{3}{2} \int_0^\infty \frac{1+z^2}{(1+z)^4} \exp[-x^2(1+z)^2] dz;$$

$$\Phi(x) = 1 \text{ at } x \ll 1; \quad \Phi(x) = (0.75/x^2) \cdot e^{-x^2} \text{ at } x \gg 1 \quad (9)$$

(some values of this function are:  $\Phi(x) = 0.80, 0.65, 0.36, 0.10, 0.0023$  for  $x = 0.1, 0.2, 0.5, 1, 2$  — see Ref. [4]).

In table 1 we give the parameters  $N_0$ ,  $E_c$  and  $\lambda_c = 2\pi\hbar c/E_c$  for the discussed colliders. For LHC all numbers for calculation are taken from Review of Particle Properties, 1994

$$N_1 = N_2 = 9 \cdot 10^7, \quad \sigma_\perp = \sigma_1 = \sigma_2 = 15 \text{ }\mu\text{m}, \quad \sigma_z = 7.5 \text{ cm}, \quad \gamma_1 = 2980,$$

and for RHIC we use the following date

$$N_{Au} = 10^9, \quad \sigma_{Au\perp} = 0.15 \text{ mm}, \quad \sigma_{Au\perp} = 11.9 \text{ cm}, \quad \gamma_{Au} = 108,$$

$$N_p = 10^{11}, \quad \sigma_{p\perp} = 0.11 \text{ mm}, \quad \sigma_{pz} = 7.2 \text{ cm}, \quad \gamma_p = 268.$$

Table 1

	LHC (PbPb)	RHIC (AuAu)	RHIC (pAu)	RHIC (pp)
$N_0$	49	600	49	170
$E_c$ (eV)	93	0.078	0.48	0.79
$\lambda_c$ ( $\mu\text{m}$ )	0.013	20	2.6	1.6

Specific features of CBS — a sharp dependence of spectrum (6) on the second bunch length, an unusual behavior of the CBS photon rate in dependence on the impact parameter between axes of the colliding bunches, an azimuthal asymmetry and polarization of photons — can be very useful for an operative control over collisions and for measuring bunch parameters.

It may be convenient for LHC to use the CBS photons in the range of *visible light*  $E_\gamma \sim 2 - 3 \text{ eV} \ll E_c = 93 \text{ eV}$ . In this region the rate of photons will be

$$\frac{dN_\gamma}{\tau} \approx 4 \cdot 10^9 \frac{dE_\gamma}{E_\gamma} \text{ photons per second} \quad (10)$$

(here  $\tau = 0.135 \text{ }\mu\text{s}$  is time between collisions of bunches at a given interaction region), and it is possible to use a polarization measurement without difficulties.

## 4 Collisions with the nonzero impact parameter of the bunches

If the first bunch axis is shifted in the transverse direction by a distance  $R$  from the second bunch axis, the luminosity  $L(R)$  (as well as the number of events for the usual reactions) decreases very quickly:

$$L(R) = L(0) \exp\left(-\frac{R^2}{2\sigma_1^2 + 2\sigma_2^2}\right). \quad (11)$$

On the contrary, for colliders with the round and identical beams the number of CBS photons decreases slowly and for small values of  $R$  this number even increases slightly — see table 2. The maximum of ratio  $dN_\gamma(R)/dN_\gamma(0)$  equals 1.06 at  $R \approx 1.5\sigma_\perp$ ; after that, this ratio decreases, but even at  $R = 8\sigma_\perp$  the number of photons decreases by a factor of only 9, whereas the luminosity decreases by 7 order of magnitude.

Table 2

$R/\sigma_\perp$	0	0.4	0.8	1.2	1.6	2	3	5	8
$dN_\gamma(R)/dN_\gamma(0)$	1	1.01	1.04	1.06	1.05	1.00	0.75	0.30	0.11

The effect does not depend on the photon energy. It can be explained in the following way.

At  $R = 0$  a considerable portion of the ions  $Z_1$  moves in the region of small impact parameters where electric and magnetic fields of the second bunch are small. With the growth of  $R$  these ions are shifted into the region where the electromagnetic field of the second bunch are larger, and, therefore, the number of emitted photons increases. For large  $R$  (at  $\sigma_\perp \ll R \ll \sigma_z$ ) fields of the second bunch are  $|\mathbf{E}| \approx |\mathbf{B}| \propto 1/R$  and, therefore,  $dN_\gamma \propto 1/R^2$ , i.e. the number of emitted photons decreases but very slowly.

This feature of CBS can be used for a fast control over impact parameters between beams (especially at the beginning of every run) and over transverse beam sizes. For the case of long bunches, such an experiment has already been performed at the SLC collider (see ref. [9]).

## 5 Azimuthal asymmetry and polarization

If the impact parameter between beams is nonzero, an azimuthal asymmetry of the CBS photons appears, which can also be used for operative control over beams. For definiteness, let the first bunch axis be shifted in the vertical direction by the distance  $R$  from the second bunch axis. When  $R$  increases, the fist bunch is shifted into the region where the electric field of the second bunch is directed almost in a vertical line. As a result, the equivalent photons (produced by the second bunch) obtain a linear polarization in the vertical direction. The mean degree of such a polarization  $l$  for the colliders with round and identical bunches is presented in table 3.

Table 3

$R/\sigma_\perp$	0	0.4	0.8	1.2	1.6	2	3	5	8
$l$	0	0.04	0.16	0.30	.44	0.55	0.75	0.91	0.97

Let us define the azimuthal asymmetry of the emitted photons by the relation

$$A = \frac{dN_\gamma(\varphi = 0) - dN_\gamma(\varphi = \pi/2)}{dN_\gamma(\varphi = 0) + dN_\gamma(\varphi = \pi/2)}, \quad (12)$$

where the azimuthal angle  $\varphi$  is measured with respect to the horizontal plane. It is not difficult to obtain that this quantity does not depend on photon energy and is equal to:

$$A = \frac{2(\gamma\theta)^2}{1 + (\gamma\theta)^4} l, \quad (13)$$

where  $\theta$  is the polar angle of the emitted photon. From table 3 one can see that when  $R$  increases, the fraction of photons emitted in the horizontal direction becomes greater than the fraction of photons emitted in the vertical direction.

If the equivalent photons have the linear polarization (and  $l$  is its mean degree), then the CBS photons get also the linear polarization in the same direction. Let  $l^{(f)}$  be the mean degree of CBS photon polarization. The ratio  $l^{(f)}/l$  varies in the interval from 0.5 to 1 when  $E_\gamma$  increases (see table 4).

Table 4

$E_\gamma/E_c$	0	0.2	0.4	0.6	0.8	1	1.5	2
$l^{(f)}/l$	0.5	0.7	0.81	0.86	0.89	0.94	0.96	0.97

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